A GLOBAL MODEL REDUCTION APPROACH FOR 3D FATIGUE CRACK GROWTH WITH CONFINED PLASTICITY

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Numerical simulation of 3D fatigue crack propagation

1. Real fatigue engineering cases are submitted to variable loading spectrum

   Crack closure effects induced by load history must be taken into account

2. Fatigue engineering cases are in the context of high number of cycles (~$10^6$ cycles !)

   The naïve approach will lead to prohibitive computation times!

Tackle complicated 3D problems: 3 main issues to solve

1. Intensive elastic-plastic computations in the bulk

2. High number of fatigue cycles (\(\sim 10^6\))

3. Crack propagation geometrical updates: remeshing and field projections?

**Objective:** reduce computational cost by several orders of magnitude
First issue: Handle the localized plasticity at crack tip

Use of an equivalent plasticity model condensed on the crack front, based on the recent work of Prof. Sylvie Pommier.

- Treat the plasticity apart from the spatial scale.
- Condense in a few scalar internal variables carried by the front the plastic behavior of the cracked structure  → Very fast!

• Consequence:

Intensive elastic-plastic computations in the bulk  →  A set of successive linear elastic problems $K(t)U(t) = F_{ext}(t)$  +  Update of the equivalent plasticity model condensed on the crack front

The equivalent plasticity model:

- Assumption on the crack propagation mechanism:
  - Crack grows because of crack tip plasticity
  - Propagation by successive plastic blunting and re-sharpening of the tip

The equivalent plasticity model:

- This propagation mechanism, suggested by Laird in 1967, links strongly the crack advance to the plastic blunting at crack tip.

In 2D, mode 1, crack advance depends on only one kinematics parameter: \( \rho \)

- we can now build the propagation law of the model:

\[
\frac{da}{dt} = \alpha \frac{d\rho}{dt}
\]

where \( a \) is the length of the crack, \( \rho \) the plastic blunting and \( \alpha \) a scalar material parameter.

The equivalent plasticity model:

- Plastic blunting is the difference of displacement between elastic and elastic-plastic cracks.

Therefore, it’s an image of the plastic state at crack tip.
The equivalent plasticity model: Displacement partitioning

• Observing FEM results yields:

At the scale of the K-dominance area, the plastic blunting can be approximated by a constant value $\rho$

• The whole elastic-plastic displacement field of the crack faces writes then:

$$ u_{\text{rep}} (r) = K_I^\infty \cdot f(\theta) \cdot \sqrt{r} + \rho $$

Where $f(\theta) \cdot \sqrt{r}$ is the classical factor given by LEFM, and $K_I^\infty$ the elastic stress intensity factor that we can easily compute.
The equivalent plasticity model:

- Write a law linking the plastic blunting variations to the elastic stress intensity factor variations

Compute very efficiently the plastic state and the crack advance from the elastic stress intensity factor

Intensive elastic-plastic computations in the bulk

A set of successive linear elastic problems

\[ K(t), U(t) = F_{ci}(t) \]

Update of the equivalent plasticity model condensed on the crack front

Application example: CT sample

- Equivalent plasticity model response to an overload:
Second issue: Reduce the cost of the numerous linear solves

- The non-linear computations are replaced by a large set of large elastic problems:

\[ K(t_k)U(t_k) = F_{ext}(t_k) \]

- \textit{Problems dimension: } \( n \approx 10^5 \) dofs
- \textit{Set dimension: } \( t_{end} \approx 10^6 \) time steps

Better, but still very time consuming!

Use a reduced basis approach to drastically decrease the computational cost of those calculations

Reduced basis approach

- Fundamental observation:

  From one propagation step to another, the problems to solve are quite similar

  The displacement solutions belong to a same low dimensional subspace

- Two steps then:

  1. Build a basis $P$ of that solution subspace
     
        $P$ is of low dimension!

  2. Search the solutions as linear combinations of the basis vectors:

     $$U(t_k) = \sum_{i=1}^{N} \lambda_i P_i$$

     \[ \text{dim } P = N \ll n \text{ dofs} \]

Reduced basis approach: incremental build of the reduced basis

The reduced basis is built incrementally, during the crack propagation

- Expansion of the basis:
  - The basis vectors are based on solved displacement solution vectors
  - When necessary, add to the basis the part of the current solution vector which is not already in there:

\[
P^{(m+1)} = \left\{ P^{(m)}, P^\perp . U(t_k) \right\}
\]

\[
\text{with } P^\perp = I - P^{(m)} . P^{(m)^T}
\]
Reduced basis approach: use of the projection operator

• A set of successive large finite element linear elastic problems:

\[ K(t_k)U(t_k) = F_{\text{ext}}(t_k) \]

Where the problems’ dimension is: \( n \approx 10^5 \) dofs

• Use a projection operator \( P \) to the low dimensional solution space:
  - Reduced displacement vector \( \tilde{u}(t_k) \):
    \[ U(t_k) = P.\tilde{u}(t_k) \]
    with \( \dim \tilde{u}(t_k) = \dim P \ll n \)
  - Reduced problem to solve:
    \[ \left[ \begin{array}{c} P^T K(t_k) P \\ \tilde{K}(t_k) \end{array} \right] \tilde{u}(t_k) = \left[ \begin{array}{c} P^T F_{\text{ext}}(t_k) \\ \tilde{F}_{\text{ext}}(t_k) \end{array} \right] \]

Straightforward computational gain: now solve a set of \( \dim P \) problems!
Third issue: Handle the geometrical updates during the propagation

• Mesh morphing definition:
  – Modify nodes coordinates of an existing mesh to fit a different geometry
  – Neither the number of nodes nor the connectivity is modified

Allows to discretize several geometries with meshes of the same dimension

reference: M. Alexa, Recent advances in mesh morphing, *Computer graphics forum* 21 (2), 173-198, Blackwell 2002
Coupling the methods: General Strategy

- The 3 different time scales and the general algorithm:

  - Propagation law scale
  - Morphing scale
  - Remeshing scale

  **Remeshing time scale**
  - Compute the corrected SIF
  - CTCP model: plastic state and crack advance

  **Morphing time scale**
  - Morph mesh to new crack front position
  - Linear elastic structural solve: Use the *a priori* model reduction
  - Compute the nominal SIF

  **Propagation law time scale**
  - Remesh the model
Remeshing versus mesh morphing approach

- Objective: estimate the dimension of the reduced basis for each approach
  - 40 propagation steps are performed without the reduced basis approach
  - Geometrical updates are handled by:

  **Node release** (equivalent to remeshing)

  **Mesh morphing**
Remeshing versus mesh morphing approach

- Objective: estimate the dimension of the reduced basis for each approach
  - Singular value decomposition of the displacement solution vectors

Matrices of the displacement solution vectors
Remeshing versus mesh morphing approach

- **Objective:** estimate the dimension of the reduced basis for each approach

*The mesh morphing technique is a key component of the reduced basis approach*
Example: mode I crack in a 2D Compact Tension sample

- **Objective:** usability and accuracy of the ROM applied to fracture mechanics
  - A reduced basis is built from 4 vector corresponding to 4 crack positions
  - The crack is grown from 10mm to 30mm by the morphing technique
Example: mode I crack in a 2D Compact Tension sample

- Objective: usability and accuracy of the ROM applied to fracture mechanics
  - Compute the stress intensity factors from both the dimension 4 ROM and the full FE solutions.
  - Compare with analytic solution: less than 0.42% of error
Example: mode I surface crack in a 3D sample under tension

- Objective: estimate the dimension of the reduced basis
  - 60 morphing steps are performed without the reduced basis approach corresponding to 264,35 loading cycles and 528,700 time steps
  - Geometrical updates are handled by mesh morphing
Example: mode I surface crack in a 3D sample under tension

- Objective: estimate the dimension of the reduced basis
  - Singular value decomposition and Gram-Schmidt orthonormalization

Matrix of the displacement solution vectors

$\begin{bmatrix}
(u_1^1) & (u_1^2) & \cdots & (u_1^{60}) \\
(u_2^1) & (u_2^2) & \cdots & (u_2^{60}) \\
\vdots & \vdots & \ddots & \vdots \\
(u_n^1) & (u_n^2) & \cdots & (u_n^{60})
\end{bmatrix}$

- $7^{th}$ singular value $< 10^{-6}$
Example: mode I surface crack in a 3D sample under tension

Shape functions of the basis built on the 5 first solution vectors

Projection error of the 60 solution vectors on the reduced basis
Example: mode I surface crack in a 3D “H” block under tension

- Objective: perform a complete analysis with the whole strategy
  - Modification of the shape of the front
  - Crack length increases by a factor 2.7 without remeshing, corresponding to 77906 loading cycles and 1558120 time steps
Example: mode I surface crack in a 3D “H” block under tension

- Objective: perform a complete analysis with the whole strategy
  - Modification of the shape of the front
  - Crack length increases by a factor 2.7 without remeshing
Example: mode I surface crack in a 3D “H” block under tension

- Reduced basis incremental enrichment during the propagation:

Among the 65 computed configurations only 1/3 are associated to a full finite element resolution
Concluding remarks

• A global strategy for 3D fatigue crack growth with crack closure effect coupling:
  – An efficient equivalent plasticity model for the whole cracked structure [Pommier 08]
  – Reduced cost of linear elastic finite element computations by using a reduced basis approach
  – Enhanced efficiency of the reduced basis strategy: geometrical updates handled by mesh morphing

• Perspectives:
  – Improve the reduced basis incremental building
  – Morphing process coupled with reduced basis approach

This approach will make possible the simulation of elastic-plastic fatigue crack propagation under variable amplitude loading within reasonable calculation time
THANKS FOR YOU ATTENTION
APPENDIX
REDUCED BASIS APPROACH
Reduced basis approach: Error estimation

- Use a projection operator $P$ to the low dimensional solution space:
  
  \[ U(t_k) = P\tilde{u}(t_k) \]

  with $\dim \tilde{u}(t_k) = \dim P \ll n$

- Reduced problem to solve:
  
  \[ \begin{bmatrix} P^T K(t_k) P \\ \tilde{K}(t_k) \end{bmatrix} \tilde{u}(t_k) = P^T F_{\text{ext}}(t_k) \]

- Error estimation:
  
  \[ \| F_{\text{ext}}(t_k) - K(t_k) \cdot (P\tilde{u}(t_k)) \| < \varepsilon \cdot \| F_{\text{ext}}(t_k) \| \]
Example: mode I surface crack in a 3D “H” block under tension

- The morphing process as a key component of the model reduction:

  The morphing process smoothes the evolution problem, allowing the model reduction approach to be efficient for 3D crack growth

Displacement solutions visualized on the moving mesh

Displacement solutions visualized on the initial mesh
Changing scale, reducing the number of degrees of freedom:

THE EQUIVALENT PLASTICITY MODEL
First issue: Handle the localized plasticity at crack tip

Use of an equivalent plasticity model condensed on the crack front, based on the recent work of Prof. Sylvie Pommier.

- Treat the plasticity apart from the spatial scale.
- Condense in a few scalar internal variables carried by the front the plastic behavior of the cracked structure → Very fast!

Consequence:

- Intensive elastic-plastic computations in the bulk

A set of successive linear elastic problems

\[ K(t)U(t) = F_{ext}(t) \]

Update of the equivalent plasticity model condensed on the crack front

Equivalent plasticity model

1. Plastic blunting is the difference of displacement between elastic and elastic-plastic cracks.

2. It is an image of the plastic state at crack tip

So, if we manage to link plastic blunting to the loading through an elastic calculation, we have written an equivalent plasticity model, condensing all the plastic displacements of the crack in only 1 scalar variable $\rho$. 
Equivalent plasticity model

- Plastic blunting vs. elastic stress intensity factor:

  E : elastic domain for the cracked structure, varies in size and position (cyclic plastic zone) analogy with isotropic and kinematics hardenings

  D : threshold for monotonic plastic zone

  M : second “elastic” domain for the cracked structure
• The identification of the model is made in 4 sub-phases:
• Fast simulation of variable loading, including structural vibrations [Hamam2006]:

- Response of the model to a single overload:
The equivalent plasticity model:

<table>
<thead>
<tr>
<th>State variables and their conjugate forces</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>State variables</strong></td>
</tr>
<tr>
<td>Elastic</td>
</tr>
<tr>
<td>Intensity of the elastic displacement field:</td>
</tr>
<tr>
<td>$K_\infty$</td>
</tr>
<tr>
<td>Crack length: $a$</td>
</tr>
<tr>
<td>Crack tip blunting: $\rho$</td>
</tr>
</tbody>
</table>

$K_\infty$ is the intensity of the elastic displacement field.
CRACK CLOSURE EFFECT
History effects and growth rates

• Crack growth rates highly depend on the loading history: This is the crack closure effects expression.

Application context and difficulties

- An incompatibility arises:

Simulate spectrum fatigue loading effects make necessary the calculation of each loading “pseudo-cycle”

It remains out of possibility to compute millions of elastic-plastic cycles on a micro spatial scale

we propose to use an equivalent plasticity model with a reduced number of degrees of freedom, based on the recent work of Prof. Sylvie Pommier.

Crack closure effect

- Crack closure occurs under fatigue crack growth conditions. It makes a part of fatigue cycles less efficient, and is mainly due to plasticity at crack tip.
- Crack tip monotonic plasticity with an applied load:
Crack closure effect

- When crack advances, plasticity makes a plastic wake, responsible for compression at crack tip and fatigue striations observable on the fracture surface.
Crack closure effect

- When crack advances, plasticity makes a plastic wake, responsible for compression at crack tip and fatigue striations observable on the fracture surface.

- It could be shown by XFEM calculation (Thomas Elguedj PhD thesis 2006)
Crack closure effect

- The residual compression stress at crack tip leads to the appearance of a load threshold to open the crack.

  - Under this threshold, the applied load cannot open the crack and initiate propagation: this is an inefficient part of the load cycle, and it is called "crack closure phenomenon"
Crack closure effect

- Overload retardation effect:

- And what about interactions between several overloads?
Application context and difficulties

- Difficulty: fatigue crack growth is a multi-scale problem

  1. In fatigue the first hypothesis is the small scale yielding implies the use of a micro scale in space.

  2. Plasticity is a path dependent phenomenon implies the use of a very fine scale in time.
Application context and difficulties

• An example: The helicopter round-robin open challenge

• An open problem initiated in 2002 by the American helicopter community to benchmark the fatigue crack growth simulation methods.
  - Complicated 3D part
  - Complicated variable loading
  - High number of cycles

BACKUP
Remeshing versus mesh morphing approach

- Objective: estimate the dimension of the reduced basis for each approach
  - Singular value decomposition of the displacement solution vectors

\[
\begin{bmatrix}
  u_1^1 \\
  u_2^1 \\
  \vdots \\
  u_n^1
\end{bmatrix}
\begin{bmatrix}
  u_1^2 \\
  u_2^2 \\
  \vdots \\
  u_n^2
\end{bmatrix}
\begin{bmatrix}
  u_1^{40} \\
  u_2^{40} \\
  \vdots \\
  u_n^{40}
\end{bmatrix}
\]

Matrix of the displacement solution vectors

The mesh morphing technique is a key component of the reduced basis approach

7th singular value < $10^{-6}$
Example: mode I surface crack in a 3D sample under tension

Shape functions of the basis built on the 5 first solution vectors

Projection error of the 60 solution vectors on the reduced basis
Example: mode I corner crack in a 3D sample under tension

- **Objective:** estimate the dimension of the reduced basis
  - Perform a singular value decomposition on the displacement solutions

Each one of the 140 solution vectors can be efficiently described by a basis of dimension 5

![Matrix of the displacement solution vectors](image)

![Singular value decomposition](image)

$5^{th}$ singular value $< 10^{-6}$
THANK YOU.